Math 143

Spring 2024

Worksheet for 10

Introduction to Parametric curves

Problem Set Instructions: Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

Trigonometry review:

(a) Draw two reference triangles, 30 - 60 - 90 and 45 - 45 - 90, and label side lengths and angles in radians.

(b) What are $\cos(\pi/4), \cos(\pi/6), \sin(\pi/6), \tan(\pi/3)$, and $\tan(11\pi/6)$?

(c) What are $\cos(5\pi/6)$,and $\cos(7\pi/6)$

(d) Find all angles θ with $\cos(\theta) = 1/2$.

(e) Find all angles θ with $sin(\theta) = -1/\sqrt{3}$.

(f) Sketch the graph in the r, θ plane of $r = \sin(2\theta)$.

(g) Sketch the graph in the r, θ plane of $r = 5 - 2\sin(\theta)$.

Recall the definition of a parametric curve: Let $x(t), y(t), a \leq t \leq b$ be functions. Then every $t \in [a, b]$ corresponds to a point $(x(t), y(t))$in the xy-plane. The curve traced out in the xy - plane for $a \leq t \leq b$is called a **parametric curve** with **initial point** $(x(a), y(a))$ and **terminal point** $(x(b), y(b))$

Parametric curves allow us to describe curves in space that don't correspond to graphs of functions.

Problems

1. Consider the parametric curve $(x(t), y(t))$, where

$$x(t) = \frac{1 - t^2}{1 + t^2} \qquad y(t) = \frac{2t}{1 + t^2}.$$

   (a) Sketch this curve as t increases from -2 to 2.

(b) What happens to x and y as $t \to \infty$. What about as $t \to -\infty$.

(c) show that this parametric curve traces out the circle $x^2 + y^2 = 1$

(d) Compute dy/dx for this curve, first by using $x^2 + y^2 = 1$, then using the parametric equations.

(e) Find an equation for the tangent line to the curve at $t = 2$.

2. Sketch the following parametric curves

   (a) $x = \sin(4\theta)$, $y = \cos(4\theta)$, $0 \leq \theta \leq \pi/2$.

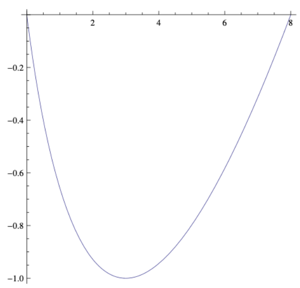
   (b) $x = t^t - t$, $y = 2^t + t$, $-2 \leq 2$.

   (c) $x = \cos^2(t)$, $y = 1 + \cos(t)$, $0 \leq \pi$.

3. Match the following parametric equations to their graphs

$x = \cos(t),\ y = \sin(t),\ 0 \le t \le \pi$	$x = \cos(3t),\ y = \sin(2t),\ 0 \le t \le 2\pi$	$x = t^2,\ y = t^5,\ -1 \le t \le 1$
$x = \cos(2t),\ y = \sin(2t),\ 0 \le t \le \pi$	$x = t + \cos(2t),\ y = t + \sin(2t),\ 0 \le t \le \pi$	$x = t^5,\ y = t^2,\ -1 \le t \le 1$
$x = 3\cos(t),\ y = 2\sin(t),\ 0 \le t \le 2\pi$	$x = 2\cos^2(t),\ y = 3\sin^2(t),\ 0 \le t \le 2\pi$	$x = t^2 - t,\ y = t^2 + t,\ 0 \le t \le 2$
$x = 2\cos(t),\ y = 3\sin(t),\ 0 \le t \le 2\pi$	$x = 1 - \sin(t),\ y = \cos^2(t),\ 0 \le t \le 2\pi$	$x = t^3 + t^2 - t,\ y = t^2 - t^4,\ -1 \le t \le 1$

Graphs: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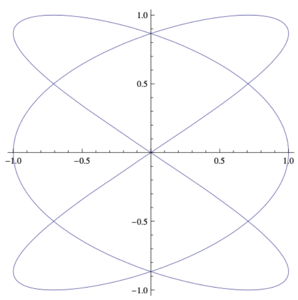
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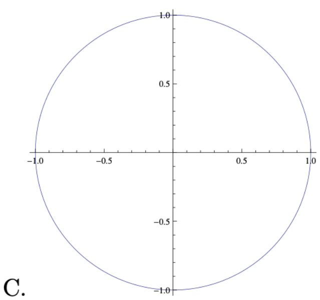
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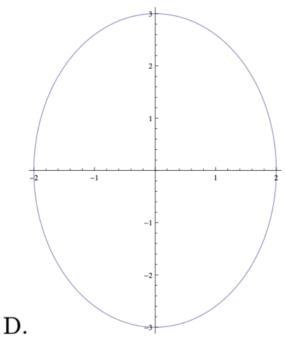
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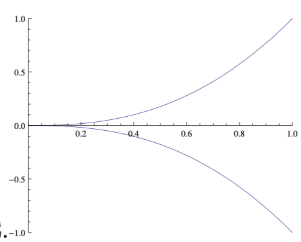
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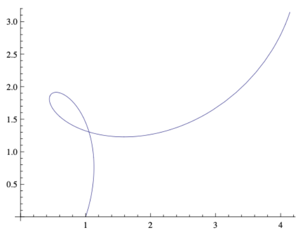
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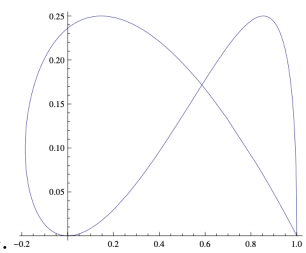
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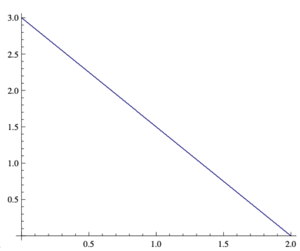
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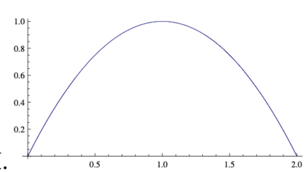
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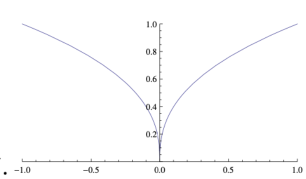
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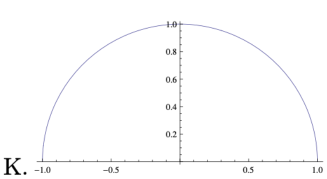
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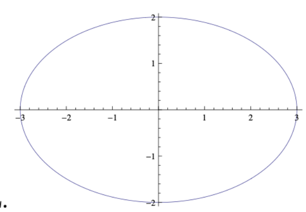
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